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SHORT NOTE

A graphical construction for shear stress on a fault surface

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Abstract—On a fault surface the orientation and sense of maximum resolved shear stress (τ) are controlled by the orientations and relative magnitudes of the principal stresses. A simple method is presented for determining the orientation and sense of τ using the orthographic projections of the greatest and least compressive stresses.

INTRODUCTION

RECENTLY, Lisle (1989), Means (1989), De Paor (1990) and Fry (1992) have described graphical methods for determining the orientation of the line of maximum resolved shear stress on a generally oriented plane. Presented here is a shear stress construction which, like those of Lisle, Means, De Paor and Fry, is simple to implement graphically. Because of its simplicity, this method may be useful in fault-slip analysis, engineering geology and other applications.

METHOD

The method of Means (1989) provides a useful starting point by decomposing a stress tensor into three components: two deviatoric components with magnitudes $(\sigma_1 - \sigma_2)$ and $(\sigma_3 - \sigma_2)$, and a hydrostatic component with magnitude (σ_2). The shear stress orientation on the fault is controlled entirely by the two deviatoric components.

Construction

To solve for the orientation of the line of maximum shear stress, first plot on a lower-hemisphere projection the fault surface great circle, the fault pole (P), and the σ_1 and σ_3 axes (Fig. 1a). Next, measure the angles between the σ_1, σ_3 axes and the fault pole, angles α and β, respectively (Fig. 1b). Then find the orthographic projections of the stress axes on the fault surface as follows: construct great circles containing the fault pole and the σ_1 axis, and the fault pole and the σ_3 axis (Fig. 1c). The intersections of the fault surface and these two planes are the orthographic projections of the σ_1 and σ_3 axes across the fault (Fig. 1c). These lines are called τ_1 and τ_3, respectively, to follow the terminology of Means (1989). Based on Means (1989), the magnitudes of the shear stresses acting along τ_1 and τ_3 are

$$\tau_1 = (\sigma_1 - \sigma_2)\cos(\alpha)\sin(\alpha)$$
$$\tau_3 = (\sigma_3 - \sigma_2)\cos(\beta)\sin(\beta).$$

In the present construction, the sense of shear in the direction of the τ_1 and τ_3 components depends on the arrangement of the fault pole, the shear stress component and the principal stress in the great circle containing these lines. When viewed in the plane containing the fault pole, the principal stress and its orthographic projection shear stress component, simple relationships exist between the arrangement of these features and the sense of displacement of the hanging wall relative to the footwall (Fig. 2). Where σ_1 has a steeper pitch than the fault pole and τ_1, it corresponds to a downward displacement of the hanging wall with respect to the footwall (Fig. 2a). Where σ_1 lies on a great circle between the fault pole and τ_1, in projection, this corresponds to an upward displacement of the hanging wall with respect to the footwall (Fig. 2b). These rules apply to σ_3 in the opposite form: where σ_3 lies on a great circle between the fault pole and τ_3, in projection, it corresponds to an upward displacement of the hanging wall with respect to the footwall (Fig. 2c). Where σ_3 has a shallower pitch than either the fault pole or τ_3, this corresponds to a downward displacement of the hanging wall with respect to the footwall (Fig. 2d).

To avoid confusion about shear sense when plotting lines τ_1 and τ_3, I use a filled circle for a downward displacement of the hanging wall, and an open circle for an upward displacement of the hanging wall. Note that in the example both τ_1 and τ_3 are downward-directed (Fig. 1c).

The next step in the method is to measure the angle (R) between τ_1 and τ_3 (Fig. 1d). Then, calculate the

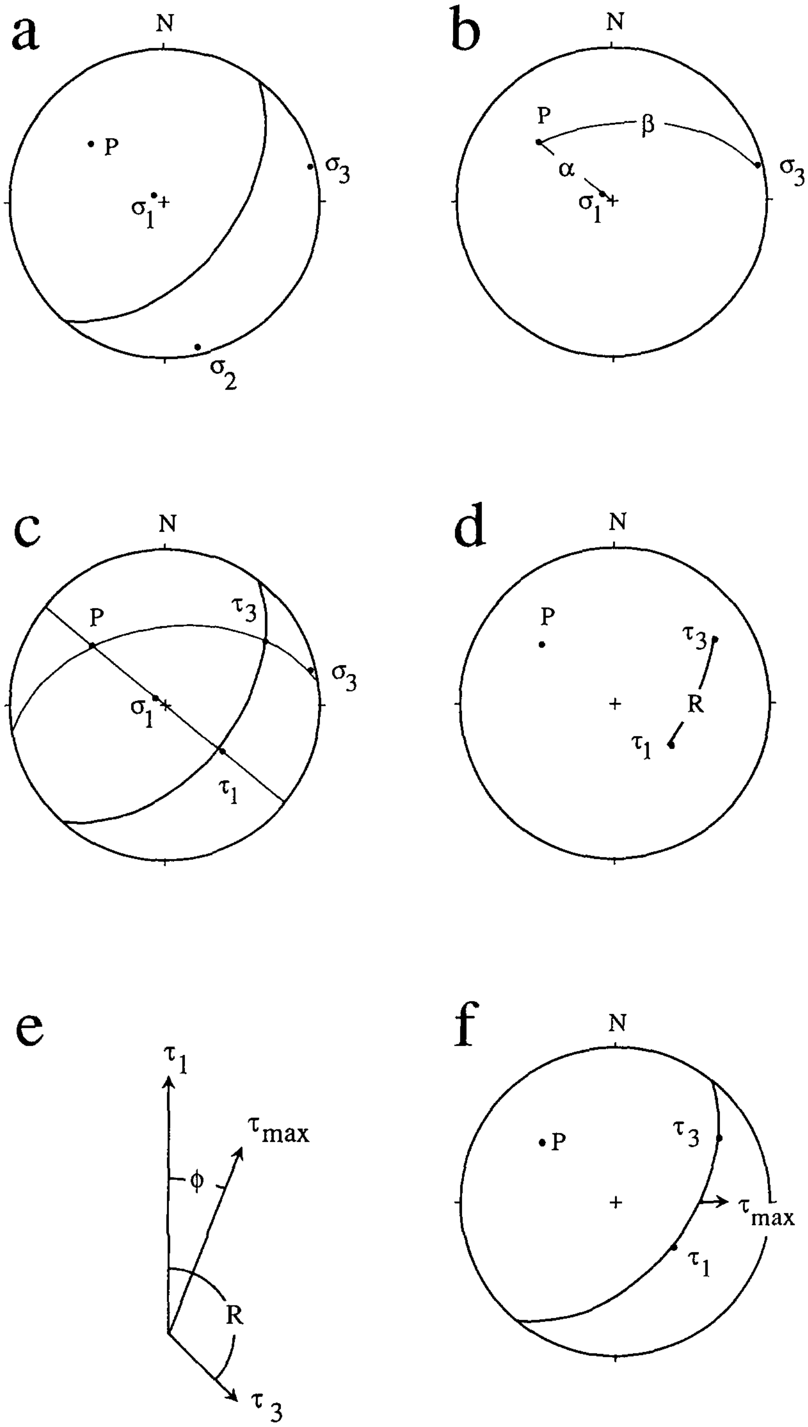

Fig. 1. Steps in the construction. (a) Orientations of the fault, the fault pole (P), and σ_1 and σ_3. The plane is oriented 039° 51° (strike and dip, right-hand rule), σ_1 plunges 84° toward 306°, σ_3 plunges 03° toward 077°. The relative magnitudes of σ_1, σ_2 and σ_3 are 1.009, 0.5616 and 0.0616, respectively. (b) Measure the angles between the fault pole (P) and stress axes σ_1 and σ_3. (c) Solve for the orthographic projections of σ_1 and σ_3 by constructing the great circles containing a stress axis and the fault pole. The intersections of these great circles with the fault surface correspond to τ_1 and τ_3. Note the senses of shear are both hanging wall-downward. Measure the angle of pitch (*d*) between τ_1 and τ_3; the angle of pitch in this example is 62°. (e) Calculation of the angle between τ_1 and the line of maximum resolved shear stress. Here, ϕ = 28°. (f) The line of maximum shear stress lies at an angle ϕ from τ_1 towards τ_3, plunges 44° towards 90°, and corresponds to an oblique-normal displacement.

angle (ϕ) between σ_1 and the line of maximum shear using

$$\phi = \arctan\left[\tau_3 \sin(\mathrm{R})/(\tau_1 + \tau_3 \cos(\mathrm{R}))\right]$$

(Fig. 1e), which combines the vector addition of τ_1 and τ_3 and the trigonometric calculation of the angle between τ_3 and the line of maximum shear (Fig. 3). The line of maximum shear stress lies at an angle ϕ from τ_1 towards τ_3 (Fig. 1f). The trend and plunge of the line of maximum shear stress can then be recovered directly from the plot. Displacement sense, hanging wall relative to footwall, is given by the polar direction of the pitch of the resolved shear sense in the fault; pitches less than 180° correspond to normal displacements in the plane of

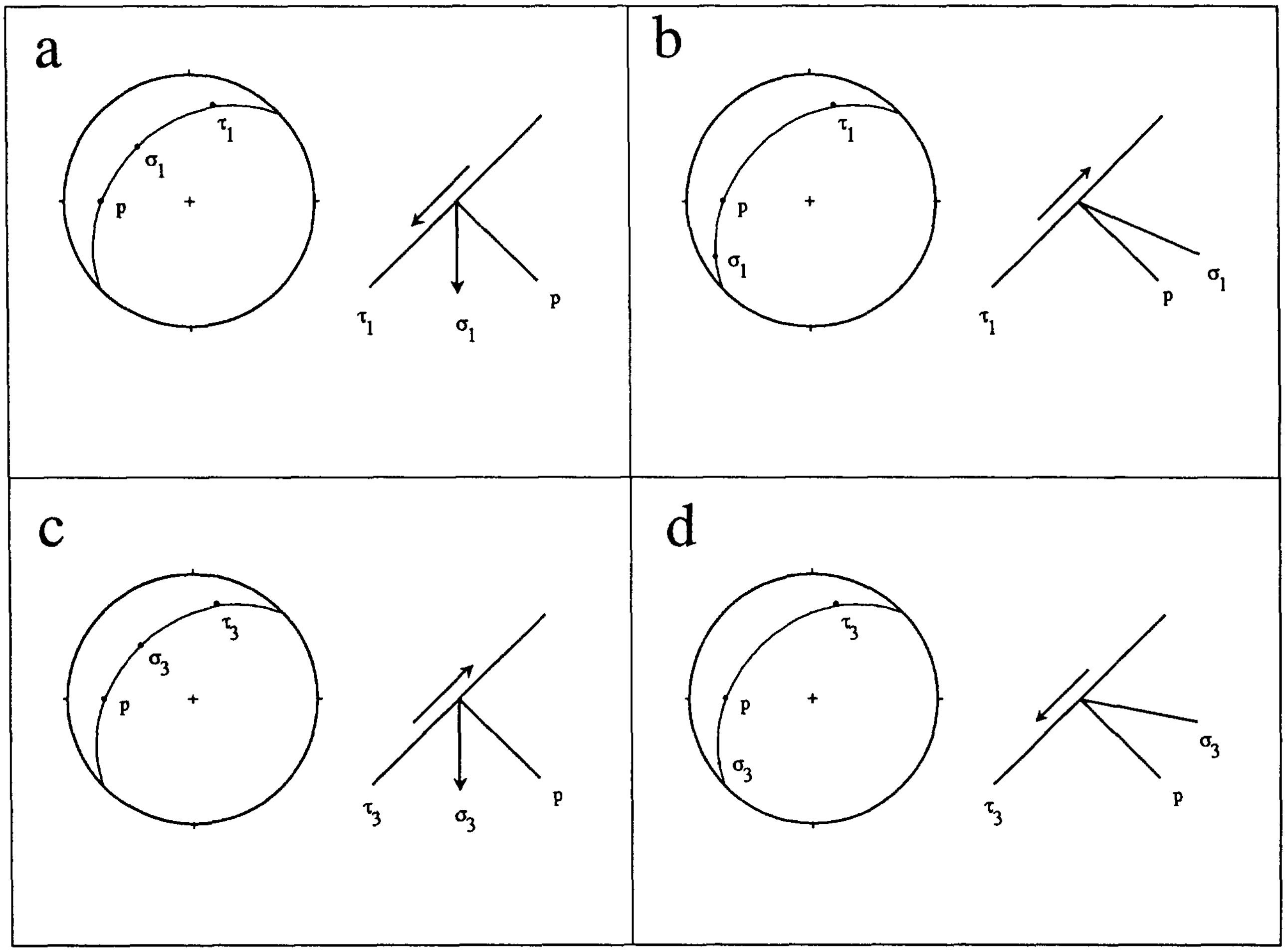

Fig. 2. Relationships between the sense of displacement of the hanging wall caused by τ_1 (a & b) and τ_3 (c & d). Great circles represent the plane containing the fault pole, principal stress and orthographic projection of the principal stress. (a) When σ_1 has a steeper pitch than both the fault pole and the orthographic projection, this corresponds to a downward displacement of the hanging wall with respect to the footwall. (b) Where σ_1 has a pitch which is less than either the fault pole or the orthographic projection, this corresponds to an upward displacement of the hanging wall with respect to the footwall. For σ_3, the relationship given in (a) and (b) above applies in reverse. (c) In cases where σ_3 has a steeper pitch than both the fault pole and the orthographic projection, this corresponds to an upward relative displacement of the hanging wall. (d) Where the pitch of either the fault pole or the orthographic projection is steeper than σ_3, this corresponds to a downward relative displacement of the hanging wall.

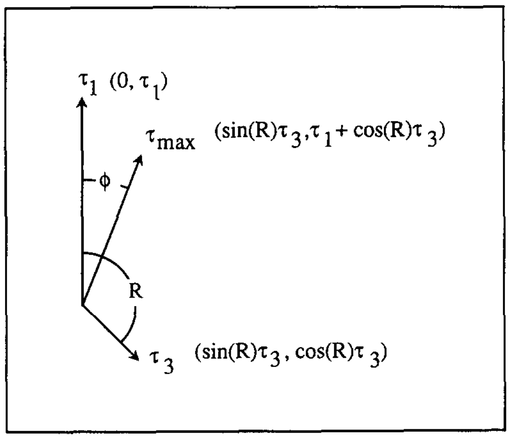

Fig. 3. Trigonometric relationships between τ_1, τ_3 and τ_{max} viewed in the plane of the fault. τ_1 is used as the y-axis, and has x and y co-ordinates (0) and (τ_1), respectively. τ_3 lies at an angle (R) from τ_1, and has x and y co-ordinates (sin (R)τ_3) and (cos (R)τ_3), respectively. Vector addition of τ_1 and τ_3 yields the vector τ_{max}, which has x co-ordinate (sin (R)τ_3) and y co-ordinate (τ_1 + cos (R)τ_3). The pitch of $\tau_{max}(\phi)$ in the fault surface is simply the arctangent of the x and y co-ordinates of τ_{max}. Similarly, the magnitude of τ_{max} is simply a function of sin (ϕ).

the projection, while pitches greater than 180° correspond to reverse displacements out of the plane of the projection. In the example provided, an oblique-normal displacement would result.

The magnitude of the resolved shear stress across the fault can be calculated by

$$\tau_{max} = (\sin(R)\tau_3)/\sin(\phi)$$

which is a replacement of a vector addition of τ_1 and τ_3 by a simple trigonometric relationship between these lines, the angle of rake between them, and the magnitude of the resolved shear stress (Fig. 3).

DISCUSSION

An advantage of this approach is that the two rotation steps found in Means (1989) are eliminated. The present method requires only elementary operations, and can easily be done on a single overlay with the aid of either a pocket calculator or personal computer. However, both methods yield the same result. This problem could also be approached using stereovectors (De Paor 1979). However, the method presented here circumvents the scaling steps required in a stereovector solution and avoids both the inaccuracy on a steeply dipping fault and the inconvenience of a change of reference frame. However, the present method does require careful construction to insure the proper interpretation of the shear stress components. This method is similar to that of Fry (1992) in finding the direction of greatest resolved shear stress as a trigonometric function of the deviatoric stress components. Fry's construction performs the operation outside of the fault plane, while the present construction performs the calculation in the fault plane. Both methods yield the same result. Likewise, this problem can be solved graphically as suggested by De Paor (1990) or numerically as suggested by Ragan (1990). The method I have presented, like that of Means (1989) from which it is derived, is simple for students' use.

One might legitimately question the need for yet another shear stress construction, especially one which bears a strong resemblance to that of Means (1989) and Fry (1992). The present method, however, has several teaching applications. First, this method serves as a vehicle for teaching the projection of a line or vector across a plane. Second, it helps develop facility working with lines which project into or out of the lower-hemisphere projection. Third, this method reinforces the idea that the direction of maximum resolved shear stress is controlled by the directions of the resolved stress components across the fault. Fourth, it has proved useful as a bridge between teaching elementary stress vector concepts, and the more sophisticated stress tensor solution for shear stress presented by Ragan (1990).

The method initially was developed to aid in instruction of the inversion of fault and slickenside data to obtain paleostress information. In this case, the initial data includes a known displacement direction and sense across a fault. Where some *a priori* estimate of the principal stress orientations can be made, a range of stress conditions can be tested using relative stress magnitudes. Gephart & Forsyth (1984) define a useful stress ratio R, where

$$R = (\sigma_2 - \sigma_1)/(\sigma_3 - \sigma_1).$$

The value of R has a numerical range of

$$0 \leq R \leq 1$$

(Gephart & Forsyth 1984). Changes in the hydrostatic component or proportional changes in stress magnitudes will not change either the value of R, or the direction of maximum resolved shear stress across a fault surface. Thus, upon completing the net construction and measuring the necessary angles, a range of stress states can be tested to predict the orientation and sense of slip on a fault surface. This is useful in that it demonstrates that both stress orientations and relative magnitudes influence the direction of maximum resolved shear stress across a fault surface. A BASIC computer program is available from the author which illustrates this calculation.

This method has proved useful both in teaching elementary graphical techniques and paleostress inversion of fault slip data. Given the simplicity of this approach, the lack of rotations, and the combination of vector and trigonometric operations, this approach may be useful in other applications as well.

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